An Information-Theoretic Basis for Designing Efficient Signal Sources Using Unstable Nonlinear Electronic Oscillators

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Abstract. We review recent progress at AMRDEC in establishing an information-theoretic basis for designing efficient signal sources using unstable nonlinear electronic oscillators. Such oscillators appear to be generally useful in ranging and communications, particularly when used in a chaotic mode. Their primary advantages lie in their tremendous natural bandwidth and simplicity of construction. We show that the dynamics of these novel devices can be modeled as an information source of discrete *symbols* with fixed transition rules. Applications involving the control and synchronization of chaotic oscillators are discussed from this vantage point.

1. INTRODUCTION

Nonlinear dynamical systems offer a completely new paradigm for building efficient, low-cost ultra-wideband electronic oscillators. The objective of this research is to develop an information theoretic foundation to nonlinear dynamical processes—particularly as a basis for controlling and synchronizing signal sources in next generation ranging and communication systems. Currently, applications of complex nonlinear phenomena such as chaos are not developed from any basic theory, but instead, are assembled from the results of *ad hoc* experiments. Consequently, there is no systematic way of engineering chaotic systems to do useful tasks or compare their potential with existing technologies. Our solution to this problem is to bridge the gap between nonlinear dynamics and communication theory. Describing nonlinear dynamics in familiar communication theory allows us to make precise statements about the applicability of nonlinear dynamics to specific problems.

2. CHAOS AND INFORMATION

The most intriguing aspect of chaotic electronic oscillators is that they can be extremely simple devices and yet produce very complex noise-like waveforms. Fig.1 shows a simple chaotic circuit based on an LC tank coupled to nonlinearity. Also shown is a representative phase portrait taken from an actual circuit. The non-repeating nature of chaotic waveforms causes them to fill in the available phase space and to be spectrally broadband. Using this architecture with lumped components one can generate a chaos with a bandwidth of hundreds of megahertz. Such waveforms have ideal thumbtack ambiguity functions, which make them well-suited for ranging (Ying, 1998).

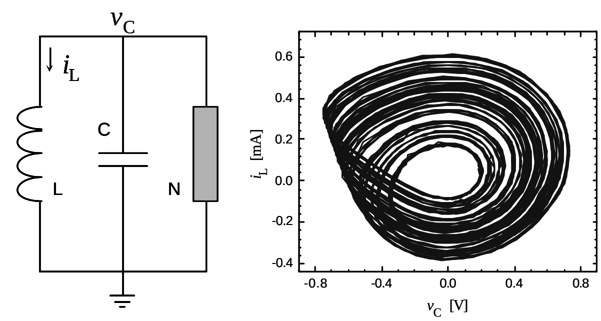

Fig. 1. High-level schematic of a simple chaotic LC circuit along with a representative phase portrait taken from an actual circuit. The 'N' denotes a nonlinear element.

A key feature of this behavior is that it exhibits nonzero Shannon entropy—that is, the waveforms contain information. We characterize this information by transforming the continuous chaotic process into an equivalent source of discrete *symbols* with certain transition rules (Collet, 1980). For systems of the type shown in Fig.1, this process is well-understood and can be implemented with a 1-bit analog to digital converter at high speed. Voltage peaks which are greater than a specially chosen threshold are labeled '1', while those below are labeled '0', as shown in Fig.2. Amazingly, this process of converting a continuous chaotic waveform into a symbol sequence is invertible. The waveform can be completely recovered from the symbols through the use of a look-up table that relates symbol sequences to voltages.

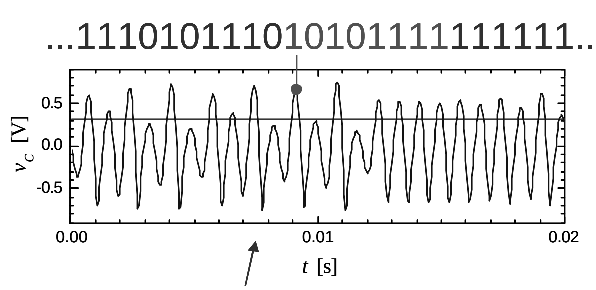

Fig. 2. Transformation of a chaotic waveform to a sequence of symbols. Peaks above the threshold are labeled '1' and those below with a '0'.

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3. CONTROLLING CHAOS

An admissible symbol sequence can be converted to a waveform through the use of a controller. Because chaos is extremely sensitive to perturbations the controller need only apply exceedingly small impulses to achieve the desired behavior (Ott, 1990). An embodiment of this process known as *dynamical limiting* is shown in Fig.3 (Corron, 2002). Symbol sequences are inputted to the sequencer which converts them to the corresponding voltage. These voltages levels are used to bias a diode which then limits the voltages swings of the chaotic oscillators. When the sequence is admissible—that is, allowed by the dynamics of the oscillator—the result is a known waveform. In this way high-bandwidth digital information can be feed directly into a chaotic oscillator and converted to a high amplitude analog waveform in an efficient manner.

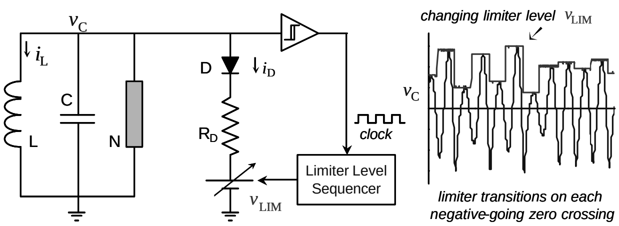

Fig. 3. Chaos control implemented using dynamic limiting. Symbols are converted to voltages in the sequencer and these voltage levels bias a diode which limits the oscillator.

3. SYNCHRONIZING CHAOS

It is well-known that chaotic oscillators can synchronize when coupled together (Pecora, 1990). This phenomenon has important applications for auto-synchronizing spread-spectrum systems, scrambling technology, and phased arrays for power-combining and beam steering. When the coupled chaotic oscillators are treated as sources of discrete symbols of information an almost exact correspondence can be made with Shannon's mathematical theory of communication (Shannon, 1949). In doing so, we were able to predict, for the first time, fundamental relationships between detector precision (Q_D), channel capacity, lag (L), and synchronization quality (Q_S). Explicitly, we found that $Q_S < Q_D + LH$, where H is the Shannon entropy of the chaotic forcing and $Q_D > H$ (Pethel, 2003). Importantly, this formula is valid even in the presence of *negative lag* ($L < 0$), i.e. the response leads the drive (Voss, 2000). Effects like these allow the possibility of beam steering in arrays of ultra-wideband emitters without the need for variable time delay elements (Corron, 2004).

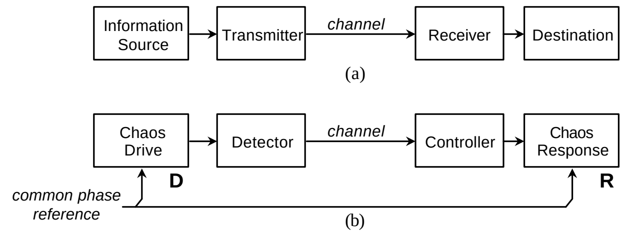

Fig. 4 a) Shannon's model of a communication system and b) the corresponding elements of unidirectional chaos synchronization.

4. CONCLUSIONS

Certain classes of chaotic oscillators can be simply transformed into an equivalent information source of discrete symbols with fixed transition rules. This allows us to catalog all the available waveforms in terms of allowed symbol sequences, and control to any of them in an optimal manner. The use of these oscillators for producing ultra-wideband radio frequency waveforms for ranging and communications can thus be put on a firm theoretical basis.

REFERENCES

Collet, P., and Eckmann, J. P., 1980: *Iterated Maps of the Interval as Dynamical Systems*, Birkhauser, Boston.

Corron, N. J., Pethel, S. D., 2002: Control of long-period orbits and arbitrary trajectories in chaotic systems using dynamic limiting, *Chaos*, **12**, 1.

Corron, N. J., Blakely, J. N., and Pethel, S. D., 2004: Beam Steering by Lag Synchronization in Wide-Bandwidth, Chaotic Arrays, *Proc. 8th Experimental Chaos Conference*, to appear.

Ott, E., Grebogi, C., and Yorke, J. A., 1990: Controlling Chaos, *Phys. Rev. Lett.* **64**, 1196-1199.

Pecora, L. M., and Carroll, T. L., 1990: Synchronization in Chaotic Systems, *Phys. Rev. Lett.* **64**, 821-824.

Pethel, S. D., Corron, N. J., Underwood, Q. R., and Myneni, K., 2003: Information Flow in Chaos Synchronization: Fundamental Tradeoffs in Precision, Delay, and Anticipation, *Phys. Rev. Lett.* **90**, 254101.

Shannon, C.E., and Weaver, W., 1949: *The Mathematical Theory of Communication*, University of Illinois Press, Urbana, Illinois.

Voss, H. U., 2000: Anticipating Chaotic Synchronization, *Phys. Rev. E* **61**, 5115-5119.

Ying, S., Weihua, S., and Guosui, L., 1998: Ambiguity Function of Chaotic Phase Modulated Radar Signals, *Proc. ICSP '98*, 1574-1577.